International Journal of Scientific & Engineering Research, Volume 8, Issue 2, February-2017
ISSN 2229-5518

Theoretical Investigation on the Steady State of Two Phase Natural Circulation Loop (NCL) with End Heat Exchangers

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Abstract

The steady state behavior of two-phase natural circulation loop with heat exchangers at the hot and cold ends is studied theoretically. A vertical rectangular loop with uniform cross section is considered along with the two concentric heat exchangers placed along the horizontal bottom and top sections of the loop. The coupling fluid is assumed to enter the hot end heat exchanger at saturation condition. Homogeneous equilibrium model is used to determine the pressure drop in the two phase sections. A one dimensional theoretical model is developed, non-dimensionalized and solved simultaneously (Momentum and energy equations) through guess and correct procedure. The model developed has been used for parametric study to determine the circulation rate of coupling fluid and exit steam quality of HEHE as a function of the non-dimensional parameters $Gr_{sat}, C_h^*, \theta_{sat}, Ntu_h, Ntu_c$, A, B and K_1.

Keywords: *Two phase, Homogeneous Equilibrium Model, Natural Circulation Loop, End Heat Exchangers.*

Nomenclature

A	$\left(\frac{\rho_{sat}}{\rho_g}-1\right)$, dimensionless
B	$\frac{(\mu_g-\mu_{sat})}{\mu_{sat}}$, dimensionless
c_{cf}	specific heat of coupling fluid at constant pressure, kJ/kg K
C_c	heat capacity rate of cold stream, kW/K
C_f	single phase friction factor, dimensionless
C_h	heat capacity rate of hot stream, kW/K
C_{cf}^*	heat capacity rate of coupling fluid, kW/K
C_c^*	non-dimensional heat capacity rate of cold stream
C_h^*	non-dimensional heat capacity rate of hot stream
C_{cf}^*	non-dimensional heat capacity rate of coupling fluid
C_{ftp}	two phase friction factor, dimensionless
$C_{min,c}$	smaller heat capacity rate of the fluid in CEHE side, kW/K
$C_{min,h}$	smaller heat capacity rate of the fluid in HEHE side, kW/K
$\left(\frac{dp}{ds}\right)$	pressure drop gradient, N/m^3
D	diameter of the loop, m
g	gravitational acceleration, m^2/s
G	mass flux, kg/m^2 s
Gr_{sat}	$\frac{g\rho_{sat}^2}{\mu_{sat}^2}$; dimensionless
h_{fg}	change of enthalpy with change of state, kJ/kg
h_{fg}^*	non-dimensional change of enthalpy with change of state, kJ/kg
K_1	L_1/L_2, dimensionless
L_1	horizontal length of the loop, m
L_2	vertical length of the loop, m
$\dot{m}$	mass flow rate of coupling fluid, kg/s
Ntu_c	$(UA)_c$ / $C_{min,c}$ (CEHE), dimensionless
Ntu_h	$(UA)_h$ / $C_{min,h}$ (HEHE), dimensionless
p	pressure, N/m^2
Re	Reynoldsnumber, dimensionless
s	space coordinate, m
T_0	reference temperature, K
T_{sat}	saturation temperature of coupling fluid, K
T_{ci}	cold stream inlet temperature, K
T_{hi}	hot stream inlet temperature, K
$(UA)_{h,c}$	product of heat transfer coefficient and heat transferarea ofHEHE and CEHE espectively, kW/K
x	mass quality, dimensionless

Greek Symbols

α	void fraction, dimensionless
ε_c	effectiveness of CEHE, dimensionless
ε_h	effectiveness of HEHE, dimensionless
θ_{ci}	non-dimensional cold stream inlet temperature
θ_{cf}	non-dimensional coupling fluid temperature

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θ_{hi} non-dimensional hot stream inlet temperature

$\bar{\mu}$ homogeneous viscosity, kg/m-s

μ_{cf} dynamic viscosity of the coupling fluid, kg/ms

μ_g gas phase viscosity of the coupling fluid, kg/ m-s

μ_{sat} coupling fluid viscosity at saturation temperature, kg/m-s

ρ_0 reference density, kg/m^3

ρ_g gas phase density of the coupling fluid/density of the gas phase, kg/m^3

ρ_m homogeneous density of the two phase, kg/m^3

ρ_{sat} coupling fluid/fluid density at saturation temperature, kg/m^3

φ angle of inclination of loop section with horizontal plane, in degrees

Subscripts

A	accelerational
c	cold stream
cf	coupling fluid
F	frictional
G	gravitational
h	hot stream

1.Introduction

In Natural Circulation Loop the loop fluid flow is driven by thermally generated density gradient. Owing to their simplicity, high heat transfer capability, and passive nature,the principle of natural circulation loops is employed in diverse engineering applications like thermo-syphon boilers, solar, thermal and waste heat recovery systems, nuclear reactors. NCLs involving evaporation and condensation of the working fluid, is specially lucrative due to large density difference of the vapor liquid phase.

Numerous investigationson two-phase NCL, both theoretical and experimental,are available in the literature [1-6]. The researchers have studied both steady state and transient performance of the loop .However the review of the literature reveals that the hot and cold ends of the natural circulation loops have been idealized either by constant temperature or constant heat flux conditions. These conditions are not appropriate when the loop exchanges heat with single phase flowing fluids. Particularly, in case of a waste recovery system, energy needs to be transferred from one fluid to another. A natural circulation loop can be conveniently employed for this purpose by incorporating suitable heat exchangers with finite heat capacity fluids at the hot and cold ends.

In the present work, the steady-state behavior of a two-phase NCL with heat exchangers at hot and cold ends has been studied theoretically based on one dimensional formulation using HEM. Various non dimensional parameters have been identified and their effect on circulation rate and steam quality is studied.

2.Natural Circulation Loop

The idealized closed rectangular two-phase NCL with end heat exchangers is depicted in .1. The loop is oriented in a vertical plane with steam and water as the working fluid. The cross section of the loop is considered to be constant and circular throughout. The hot end heat exchanger (HEHE) and the cold end heat exchanger (CEHE) are placed along the horizontal bottom and top sections of the loop, respectively. The type of heat exchangers considered in the present study is concentric-tube heat exchanger. It is assumed that in the loop, fluid flows in counter clock-wise direction. The working of the system at steady state can be explained as follows. The loop fluid enters the HEHE at temperature T_{sat} and density ρ_{sat}(x=0). It absorbs the latent heat from the hot stream while passing through the HEHE and becomes two-phase(x=x). The two phase fluid rises up under the influence of buoyancy force through the adiabatic riser (N-O). While flowing through the CEHE the fluid rejects heat to the sink and finally attains the saturation temperature T_{sat} and density ρ_{sat}. The downward flow through the adiabatic down comer (P-M) completes the circulation of the loop. The mass fraction at the exit of HEHE and the circulation rate of the loop fluid are the unknown parameters.

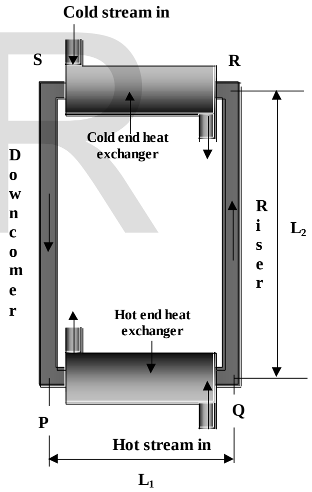

Fig.1 Schematic diagram of a Two-Phase NCL with end heat exchangers

The analysis of the loop has been made based on following assumptions.

- The loop operates under steady state and the flow is one dimensional.
- The hot and cold streamsare in single phase with constant heat capacity rates.
- The overall heat transfer coefficients and the surface area per unit length of the heat exchangers are constant throughout their lengths.
- The thermal properties of all fluid streams are uniform and constant.
- The hot stream, cold stream and the coupling fluid inside the loop are in single phase.
- Axial conduction and viscous dissipation effects are neglected.

- Minor losses due to bends and fittings have been neglected.

3. Mathematical Model

Due to the presence of two phases at different parts of the loop, momentum equation can be obtained as follows by considering frictional, accelerational and gravitational pressure drops.

$$-\frac{dp}{ds}=\left(\frac{dp}{ds}\right)_F+\left(\frac{dp}{ds}\right)_G+\left(\frac{dp}{ds}\right)_A \qquad (1)$$

For single phasethe frictional, accelerational and gravitational pressure dropscan be written as

$$\left(\frac{dp}{ds}\right)_F=-\frac{2C_f G^2}{D\rho_{sat}},\left(\frac{dp}{ds}\right)_A=-G^2\frac{d}{dz}\left(\frac{1}{\rho_{sat}}\right) \text{ and } \left(\frac{dp}{ds}\right)_G=-\rho_{sat} g\sin\varphi \qquad (2)$$

By Homogeneous Equilibrium Model (HEM), the frictional, accelerational and gravitational pressure

$$\frac{a2^{5-2b}\dot{m}^{2-b}}{\pi^{2-b}\mu_{sat}{}^{-b}D^{5-b}\rho_{sat}}\left\{\begin{array}{l} 2\left(L_1+L_2\right)+x\,(A+Bb)\,(L_1+L_2)+ \\ x^2\left[\frac{b\,(b-1)\,B^2+2AB}{6}\right](2L_1+3L_2)+ \\ x^3\left[\frac{b\,(b-1)\,(b-2)\,B^3+3b\,(b-1)\,AB^2}{12}\right](L_1+2L_2)+ \\ x^4\left[\frac{b(b-1)\,(b-2)\,(b-3)B^4+4b\,\,(b-1)\,(b-2)\,AB^3}{120}\right](2L_1+5L_2)+ \\ x^5\left[\frac{b\,(b-1)\,(b-2)\,(b-3)\,(b-4)\,B^5+b\,(b-1)\,(b-2)\,(b-3)\,AB^4}{360}\right](L_1+3L_2) \end{array}\right\}$$

$$=g\rho_{sat}\frac{A}{(1+Ax)}L_2$$

drops for two-phase can be written as

$$\left(\frac{dp}{ds}\right)_F=-\frac{2C_{ftp}G^2}{D\rho_m},$$

$$\left(\frac{dp}{ds}\right)_A=-G^2\frac{d}{ds}\left[\frac{x^2}{\rho_g\alpha}+\frac{(1-x)^2}{\rho_{sat}(1-\alpha)}\right],$$

$$\text{and } \left(\frac{dp}{ds}\right)_G=-\rho_m g\sin\varphi \qquad (3)$$

The friction factor C_{ftp} has been evaluated by using a mean two-phase viscosity $\overline{\mu}$ in the normal friction factor-Reynolds number relationship[7]

$$C_{ftp}=a\left(\frac{GD}{\overline{\mu}}\right)^{-b} \qquad (4)$$

Following relationship between x and $\overline{\mu}$ proposed by Cicchittile et al. [8] has been used.

$$\overline{\mu}=\mu_{sat}\left(1+x\frac{\left(\mu_g-\mu_{sat}\right)}{\mu_{sat}}\right) \qquad (5)$$

Substitution of Eq. (5) into Eq. (4) gives rise to

$$C_{ftp}=a\,\mathrm{Re}^{-b}\left(1+Bx\right)^b. \qquad (6)$$

$$\text{Where, } B=\frac{\left(\mu_g-\mu_{sat}\right)}{\mu_{sat}}$$

The two phase density can be written as [7], [9]

$$\rho_m=\left\{\alpha\rho_g+\left(1-\alpha\right)\rho_{sat}\right\} \qquad (7)$$

The relation between void fraction ‘α’ and mass quality ‘x’ is given by the following equation [7]

$$\rho_m=\frac{\rho_{sat}\left(1-\alpha\right)}{\left(1-x\right)} \qquad (8)$$

Substitution of Eq.(8) into Eq.(9) gives rise to

$$\rho_m=\frac{\rho_{sat}}{\left[1+Ax\right]} \qquad (9)$$

$$\text{where } A=\left(\frac{\rho_{sat}}{\rho_g}-1\right)$$

For a linear change of mass quality ‘x’ over a heating length ‘L_1’

$$\frac{dx}{ds}=\frac{x}{L_1}=\text{constant}. \qquad (10)$$

The accelerational pressure drop and compressibility of the gaseous phase has been neglected. Binomial series is used for the expansion of function $(1+Bx)^b$ and higher order terms are neglected.Following simplified loop momentum equation is obtained by integrating the pressure drops over different sections of the loop

Eqn.10 can be non-dimensionalized by using the following substitutions.

$$N=\frac{\pi^{2-b}}{a\,2^{5-2b}} \qquad (11)$$

$$Gr_{sat}=\frac{g\rho_{sat}{}^2}{\mu_{sat}{}^2} \qquad (12)$$

$$K_1=\frac{L_1}{L_2} \qquad (13)$$

$$C_{cf}{}^*=\frac{C_{cf}}{(\pi\,c\,D)_{cf}} \qquad (14)$$

The non dimensional momentum equation can be expressed in terms of two important variables of the loop namely $C_{cf}{}^*$ and x, as,

$$Cc_f^{\ *} = N\,Gr_{sat}\,\frac{A}{(1+Ax)}\left\{\begin{array}{l} 2\left(K_1+1\right)+x\;(A+Bb)\,(K_1+1)+ \\ x^2\left[\frac{b\,(b-1)\,B^2}{2}+ABb\right]\left(\frac{2K_1}{3}+1\right)+ \\ x^3\left[\frac{b\,(b-1)\,(b-2)B^3}{6}+\frac{b\,(b-1)\,AB^2}{2}\right]\left(\frac{K_1}{2}+1\right)+ \\ x^4\left[\frac{b\,(b-1)\,(b-2)\,(b-3)B^4}{24}+\frac{b\,(b-1)\,(b-2)\,AB^3}{6}\right]\left(\frac{2K_1}{5}+1\right)+ \\ x^5\left[\frac{b\,(b-1)\,(b-2)\,(b-3)\,(b-4)B^5}{120}+\frac{b\,(b-1)\,(b-2)\,(b-3)\,AB^4}{24}\right]\left(\frac{K_1}{3}+1\right) \end{array}\right\} \quad (15)$$

The energy interactions in the HEHE and CEHE can be expressed as follows.

$$\dot{m}\,x\,h_{fg} = \varepsilon_h C_{\min,h}(T_{hi} - T_{sat}) \quad (16)$$

$$\dot{m}\,x\,h_{fg} = \varepsilon_c C_{\min,c}(T_{sat} - T_{ci}) \quad (17)$$

Where,for condensationand evaporation,

$\varepsilon_{h,\,c} = 1 - e^{-Ntu_{h,\,c}}$, $C_{\min,h} = C_h$ and $C_{\min,c} = C_c$, [10]

We define the following Non-Dimensional numbers

$$\theta_{h,c,sat} = \frac{T_{h,c,sat} - T_{ci}}{T_0 - T_{ci}} \quad (18)$$

Where, T_0 is the reference temperature. For the present problem the reference temperature T_0 is equal to the inlet temperature of the hot stream (T_{hi}).

$$C_{h,\,c,\,cf}{}^{*} = \frac{C_{h,\,c,\,cf}}{(\mu c D)_{cf}} \quad (19)$$

$$Ntu_{h,c} = \frac{(UA)_{h,c}}{C_{\min,h,c}} \quad (20)$$

$$h_{fg}^{\ *} = \frac{h_{fg}}{c_{cf}\ (T_0 - T_{ci})} \quad (21)$$

Now, the non dimensional energy equations can also be expressed in terms of $Cc_f^{\ *}$ and x, as,

$$C_{cf}^{\ *} = \frac{\varepsilon_h C_h^{\ *}(\theta_{hi} - \theta_{sat})}{x h_{fg}^{\ *}} \quad (22)$$

$$C_{cf}^{\ *} = \frac{\varepsilon_c C_c^{\ *}(\theta_{sat} - \theta_{ci})}{x h_{fg}^{\ *}} \quad (23)$$

As $\theta_{hi} = 1.0$ and $\theta_{ci} = 0$, Eq.(22) and (23) can be simplified as follows.

$$C_{cf}^{\ *} = \frac{\varepsilon_h C_h^{\ *}(1 - \theta_{sat})}{x h_{fg}^{\ *}} \quad (24)$$

$$C_{cf}^{\ *} = \frac{\varepsilon_c C_c^{\ *}\theta_{sat}}{x h_{fg}^{\ *}} \quad (25)$$

4. Solution procedure

The unknown parameters, namely, the mass fraction at the exit of HEHE (x) and the circulation rate of the loop fluid ($C_{cf}^{\ *}$) can be determined by solving Eq. (15), (24) and (25) simultaneously. Since these are coupled equationsa suitable guess and correct numerical procedure is used to solve them.

5. Conclusion

Theoretical investigation ofone-dimensional steady state-analysis of the two-phase Natural Circulation Loop with heat exchangers at hot and cold end has been presented. Homogeneous equilibrium model has been used to predict the two-phase friction factor. The relevant non-dimensional numbers for the loop performance have been identified and the loop momentum equation and energy equations have been written in non-dimensional form. The mathematical model developed has been used for parameter study to determine the circulation rate of the loop and the exit quality of hot end heat exchanger as a function of the non dimensional parameters, namely, $Gr_{sat}, C_h^{\ *}, \theta_{sat}$, Ntu_h, Ntu_c, A, B and K_1.

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